Faster than the Speed of Light

Special Relativity

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Abstract

On September 23rd 2011 the OPERA experiment, which observes a neutrino beam, presented results in a seminar at CERN. The result is based on the observation that the neutrinos appear to travel at a velocity 20 parts per million above the speed of light. [1] Should this be confirmed true, it is likely to have far reaching consequences for Theoretic Physics. A similar anomalous result was reported when neutrinos from SN 1987A reached Earth three hours before visible light. It is, however, argued in this report that *Special Relativity* remains a viable physical theory even if such a result is confirmed true.

1 Introduction

The speed of light, denoted as c, is the universal upper limit to all signal speeds and plays a fundamental role in theoretical physics. Special Relativity tells us, for example, that

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{1}$$

and therefore, no application of any finite force for any finite amount of time will ever achieve $v > c$. The relativistic energy of a particle is given by Einstein's famous equation $E = mc^2$. Using (1) we can re-write this as

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{2}$$

We have that particles also satisfy

$$E^2 = p^2c^2 + m^2c^4 \tag{3}$$

Claims that it is possible to travel faster than the speed of light are not new. Tachyons are proposed hypothetical particles which always travel faster that c. Further, the Scharnhorst effect predicts that faster than c travel is possible, however it has yet to be confirmed experimentally. We shall consider what it means for Special Relativity if the results from the CERN-OPERA experiment are confirmed true, with a particular focus on the principle of causality.

1.1 Neutrino properties

Before considering the theoretical consequences of such a result it is important that we understand the properties of neutrinos. Neutrinos form a fundamental part of the Standard Model of elementary particles, which tells us that all matter is constructed from 12 fermions: 6 quarks and 6 leptons. [11] They are electronically neutral and effected only by the weak sub-atomic force. Therefore, neutrinos are able to travel large distances through matter almost uneffected. There exist three types, or "flavours", of neutrinos: electron neutrinos, muon neutrinos and tau neutrinos. Each of these also has a corresponding antiparticle.

In the Standard Model, the mass of all "flavours" of neutrinos are assumed to be zero. However, the theory of neutrino oscillation, implies that neutrino's must have a small non-zero mass. [11] Therefore, returning to our equation (1), if neutrino's have mass, we hypothesise that neutrino's must travel at a speed slightly below c.

2 Scharnhorst effect

The *Scharnhorst effect* predicts that in a vacuum with perfectly reflecting boundaries, photons can travel at a speed slightly larger than c. [3] Conventionally when one thinks of a vacuum they consider it to be empty space, void of any matter. Quantum electrodynamics (QED), however, predicts that a vacuum, far from being empty, contains pairs of electrons and positrons, which appear and disappear within short time intervals. These are known as vacuum fluctuations and are permitted by Heisenberg's uncertainty principle. [4]

The electrons and positrons can either exchange a "virtual" photon or alternatively, one of the particles may emit a "virtual" photon and then reabsorb it. This is known as a *two-loop process*. When a photon travels through a vacuum they create virtual electron-positron pairs which quickly annihilate to leave a photon. The two-loop processes which take place contribute to the speed we measure for light in a vacuum.

When one places metal plates close together they are able to exclude all virtual photons which have wavelengths larger than twice the distance between the plates. This is known as the *Casimir effect* and results in a force that pulls together the two metal plates. This means that when the photons travel between the plates, they spend a smaller fraction of their flight time undergoing two-loop processes than they would in a vacuum and so would travel marginally faster.

Theoretically, for photons travelling between plates separated by 1 micrometre, the speed of light should be increased by about one part in 10^{36} over its value in the vacuum. However, this effect is considered to be too small to be measured experimentally. [5]

Given the large difference in magnitude between the results obtained by the CERN-OPERA and those predicted by the Scharnhorst effect, it's unlikely that this could account for the OPERA results. Furthermore, the Scharnhorst effect specifically applies to photons rather than neutrinos. Potentially, however, the Scharnhorst effect could have far reaching consequences with regards to the principle of causality as discussed in Section 5.

3 Neutrino results

We will focus the attention of our report on two independent results, both of which seem to suggest that neutrinos are able to travel at a speed which is greater than c; the CERN-OPERA experiment and SN 1987A.

3.1 The OPERA neutrino velocity result

The CERN-OPERA experiment yields results that neutrinos appear to travel at a velocity 20 parts per million above the speed of light. This corresponds to a time of flight shorter by $60 \pm 6.9(\text{stat.}) \pm 7.4(\text{sys.})$ ns than that expected if they were to travel at a speed of light c. [7]

3.1.1 Criticisms of the method

The time of flight between where the neutrinos are generated at CERN and where they are received at the Laboratorio Nazionale del Gran Sasso (LNGS) is established by time stamping local reference clocks at both sites using a single, common view, GPS clock. [7] This can be referred to as one-way speed of light measurement. By comparison two-way speed of a light measurement uses just one clock and involves reflecting the light back to the origin. This avoids the difficulty in synchronising the time signals.

Imperial College Professor, Carlo R. Contaldi's paper [7], provides an argument that there exists sufficient ambiguity in the synchronisation of the two clocks used in the CERN-OPERA that the result obtained may not be interpreted as they have been. Rather, one should expect a time of flight differing slightly from c, given the experimental set-up.

Independent replication of the CERN-OPERA neutrino experiment would be needed before any conclusions from these anomalous results could be drawn.

3.2 Supernova 1987A

Approximately three hours before the visible light from SN 1987A reached Earth, a burst of neutrinos were detected. This could suggest that neutrinos are able to travel at a speed which is faster than the speed of light and potentially could provide a verification that the results described at the CERN-OPERA experiment are indeed correct.

3.2.1 Alternative explanation

It has been proposed that the discrepancy could be attributed to the interesting properties of neutrinos. Based on our current knowledge of supernovas, the detected neutrinos from SN 1987A were produced in the initial stellar collapse. However, the emission of visible light only occurs after the shock wave reaches the stellar surface. [9] Neutrinos experience limited interaction with matter and are uneffected by either the strong nuclear force or electromagnetism. Visible light, however, is a form of electromagnetic radiation. Therefore, the photons are likely to have interacted with the electromagnetically charged matter within the supernova. This could have resulted in the photons being contained within the interior of the supernova for approximately three hours, after which the photons were able to escape and travel towards Earth. It has been suggested that this could have caused the observed time gap between the neutrino and optical detections of the event. [8]

Unlike the CERN-OPERA experiment, replication of these results is impossible. Further, one is not able to verify whether the photons were indeed emitted from the same place and at the same time as the neutrinos. Hence, it is difficult to draw any firm conclusion as to whether the neutrinos did indeed travel faster than the speed of light.

3.3 Comparison between the two experimental results

The relative difference between the velocity of the neutrinos, v, with respect to the speed of light, is quoted by OPERA as $\frac{v-c}{c} = 2.48 \pm 0.28(\text{stat.}) \pm 0.30(\text{sys.}) \times 10^{-5}$.

[10]

SN 1987A is approximately $L = 1.68 \times 10^5$ light years away from Earth. If we were to assume that the neutrinos emitted from the supernova travel at the speed predicted by OPERA we would expect the neutrinos to have reached Earth approximately 4 years before visible light. [10] Given that the neutrinos only arrived 3 hours before the visible light, we can therefore argue that the two results are inconsistent.

4 Tachyon

A tachyon is a hypothetical particle which is able to travel faster than the speed of light. [6] They possess the property that when they gain energy, they slow down, with c the slowest speed they are able to attain. If we consider expression (2), we define $m_0 = iz$, where z is a real number, and thus obtain

$$E = \frac{zc^2}{\sqrt{\frac{v^2}{c^2} - 1}} \tag{4}$$

Therefore, $E \to 0$ as $v \to \infty$. Tachyons therefore satisfy the dispersion relation

$$E^2 = p^2c^2 - m^2c^4 \tag{5}$$

A tachyonic particle of mass zc^2 and energy E then travels faster than c by an amount equal to [10]

$$\frac{v - c}{c} = \frac{c}{c + v}\left(\frac{zc^2}{E}\right)^2 \tag{6}$$

Potentially, one could propose that a neutrino is a tachyon. This could therefore, provide a possible, plausible explanation of the OPERA data.

4.1 Could a neutrino be a tachyon?

We shall consider whether a neutrino, is in fact a tachyon. The CERN experiment involved neutrino's travelling a distance $L \approx 730$ km. Their associated early arrival time is given as $\delta t = \frac{L}{c}\frac{v-c}{c}$. [10]

The OPERA experiment considers two neutrino beams with *mean* energy equal to $E_1 = 13.8$ GeV and $E_2 = 40.7$ GeV respectively. [1] If we consider expression (6) we obtain

$$\frac{\delta t_1}{\delta t_2} = \left(\frac{E_2}{E_1}\right)^2 \tag{7}$$

Therefore, for $E_1 = 13.8$ GeV and $E_2 = 40.7$ GeV, one would expect

$$\frac{\delta t_1}{\delta t_2} \approx 8.7$$

However, the results obtained were $\delta t_1 = 54.7 \pm 18.4(\text{stat.}) \pm 7.1(\text{sys.})$ ns and $\delta t_2 = 68.1 \pm 19.1(\text{stat.}) \pm 7.1(\text{sys.})$ ns. Therefore, the experimental evidence indicates that

$$\frac{\delta t_1}{\delta t_2} \approx 0.80$$

which provides some indication that a neutrino is not a tachyon. However, we should note that the energies quoted by OPERA are *mean* energies and therefore we may hesitate to draw any firm conclusions without first considering independent verification.

5 Causality

Causality is a fundamental principle of theoretical physics. In essence, we might describe one event as the cause of another. For example, if event K_1 represents pulling the trigger of a loaded gun and event K_2 represents the gun firing, we would describe K_1 as the cause of K_2. The order of these two events is important. Not only does event K_1 occur earlier in time than K_2, but if we were to reverse the order of the events it isn't conceptually possible for event K_2 to be the "cause" of event K_1.

5.1 Tachyon and causal paradoxes

If a particle is able to travel at a speed faster than c, then we can generate causal paradoxes. This occurs because if we have two events say K_1 and K_2 then there is no absolute time ordering between them. Thus, for example, if a signal travels from K_1 to K_2 in a reference frame, then it is *always* possible to find a reference frame where K_1 and K_2 occur simultaneously and a reference frame where K_2 occurs before K_1. In the second frame it would appear like the signal is travelling at infinite speed, and in the third reference frame it would appear like the signal is "travelling to the past". [3]

5.1.1 Tachyonic anti-telephone

If it were possible to send a signal at a speed greater than c this could potentially create a paradox, since one could theoretically be able send a message to one's own past. This problem is sometimes referred to as the *tachyonic anti-telephone.* [3] Let's consider a situation where in some inertial frame Σ a tachyon is emitted at $t_0 = 0$, $x_0 = 0$, denoted as K_0 in Figure 1. This signal is received at K_1 at some time $t_1 > 0$. At K_1 a second tachyon is emitted which is received at K_2, which occurs at some future time with respect to inertial frame Σ' and to the past with respect to inertial frame Σ. Now, if events are constructed such that K_0 causes K_1 which in turn causes K_2, this means event K_0 causes K_2. This leads to a paradox since $t_0 > t_2$; K_0 follows event K_2.

For example, if event K_0 represents person A sending a message to person B and event K_1 represents person B receiving A's message and sending a response

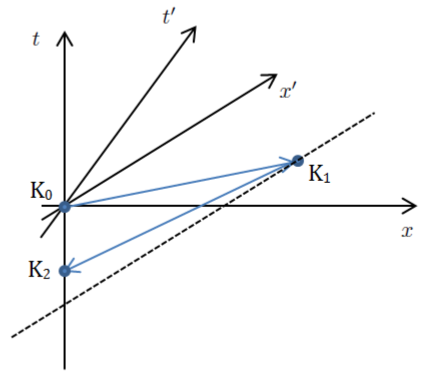

Figure 1: A tachyon causal paradox. The dotted line represents the set of events which are simultaneous with K_1 in reference frame $\Sigma^{'}$.

back to person A, then person A receives the response from B before they have even sent their message.

We were to elaborate this with the following example. Suppose that A and B are married and at time t_n such that $t_0 > t_n > t_2$, A and B conceive a child. At event K_0 A sends a message to B telling him that he is to become a father. B then promptly sends a message back to A telling her that he wishes to break-up. This message is received at a time t_2 which is before t_n and therefore A and B would never have been together to conceive a child in the first place, leading to a paradox. A similar famous example is the so called *godfather paradox*. Here we suppose time travel is possible and that a man goes back in time and kills his grandfather before he met his grandmother. Therefore, the man could have never have been born and therefore he could have never have gone back in time to kill his grandfather. This leads to a paradox. [12]

5.2 Are the existence of Tachyons consistent with Special Relativity

We shall show that despite the above problems, the existence of Tachyon's still remain consistent with Special Relativity, when given appropriate consideration. We start with the relation

$$c\frac{d}{ds} = \gamma\frac{d}{dt} \tag{8}$$

where $\gamma = \frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}$.

We shall now denote v as the velocity of some object and V as the velocity of the reference frame. We shall consider the case where $V < c$ and $v > c$. From (8) we obtain

$$(ds)^2 = \left(1 - \frac{v^2}{c^2}\right)(dt)^2 \tag{9}$$

which is negative for $v > c$. The Lorentz transformation between two frames are given by

$$dx' = \left(1 - \tfrac{V}{v}\right)\gamma dx \qquad dt' = \left(1 - \tfrac{vV}{c^2}\right)\gamma dt$$

Therefore, if we choose our frame of reference such that $vV > c^2$, then dt' becomes negative. This creates the problem of time travel.

However, one is able to reformulate the problem using the *Feinberg reinterpretation principle*, to try and avoid the problem of causality. We note that the energy E also undergoes a Lorentz transformation such that

$$E' = \left(1 - \frac{vV}{c^2}\right)E$$

For the case of $dt' > 0$, we consider a process which emits a particle of energy $+|E|$ at t'_1 which is then absorbed at the later time t'_2. If we now consider the case then dt' is such that $dt' < 0$. Let's suppose that at a time t'_2 a particle of energy $-|E|$ is emitted which is absorbed at the earlier time t'_1. We now reinterpret this event as the emission of an anti-particle at t'_1 and its subsequent absorption at t'_2.

Thus one is able to avoid such casual paradox's and demonstrate that the existence of a Tachyon is in fact consistent with Special Relativity. [12]

6 Conclusion

In conclusion, we have provided an alternative explanation which may account for the results obtained in the CERN-OPERA experiment, citing in particular the problems caused by one-way speed of light measurements. Further, based on our current understanding of supernovas, neutrinos from SN 1987A arriving before visible light should be expected. We have also demonstrated that the results from the two situations are inconsistent. We therefore, conclude that there is not sufficient evidence to claim that neutrino's travel faster than c.

However, we have described how the Scharnhorst effect means that it is theoretically possible for photons to travel faster than c. We have described some of the causal implications of faster than c communication, however have shown that using the Feinberg reinterpretation principle, one is able to avoid such causal paradoxes.

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